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Quantum Anomalies in Dense Matter

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We consider the effects of quantum anomalies involving the baryon current for high-density matter. In the effective Lagrangian, the anomaly terms describe the interaction of three light fields: the electromagnetic photons A_μ, neutral light Nambu-Goldstone bosons (π, η, η'), and the superfluid phonon. The anomaly induced interactions lead to a number of interesting phenomena which may have phenomenological consequences observable in neutron stars.

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Introduction.—The rich phase diagram of QCD at high baryon density has attracted considerable attention recently [1]. Much of attention has been paid to the determination of possible symmetry breaking patterns, which are important for understanding the low-energy dynamics. A typical example is QCD with three light flavors. The ground state [the color-flavor-locked (CFL) phase] was determined to break baryon number, chiral and $\mathrm{U}(1)_\mathrm{A}$ symmetries. One then writes down an effective Lagrangian and find it parameters by matching calculations.

On the other hand, effects originating from quantum anomalies have not been systematically studied (exceptions include Refs. [2, 3]). It is well known that anomalies have important implications for low-energy physics: the electromagnetic decay of neutral pions is a textbook example. In this Letter we investigate the roles of anomalies at finite density. In contrast to previous works, we concentrate on the anomalies involving the baryon current. The effects coming from these anomalies are strikingly unusual; they reveal intricate interactions between the topological objects such as vortices and domain walls, the Nambu-Goldstone (NG) bosons and gauge fields. We mention here only two effects: (i) there are classical weak neutral currents flowing on superfluid vortices; (ii) there are electric currents flowing on $\mathrm{U}(1)_\mathrm{A}$ vortices, which exist at high energies [4]. More effects are considered in the paper.

Anomalies.—One method to derive anomalous terms in effective theories is to put the system in external background gauge fields and require that the effective theory reproduces the anomalies of the microscopic theory. We thus consider QCD in the background of two U(1) fields: the electromagnetic field A_μ and a fictitious (spurion) B_μ field which couples to the baryon current. Only at the end of the calculations we will put $B_\mu = \mu n_\mu$, $n_\mu = (1, \vec{0})$, corresponding to a finite baryon chemical potential. (The technique resembles the one used in Refs. [5, 6] in a similar context.) The Lagrangian describing the coupling of quarks with B_ν is

$$\mathscr{L}_\mathrm{B} = \bar{\psi}\gamma^\nu \left(i\partial_\nu + \tfrac{1}{3}B_\nu\right)\psi \tag{1}$$

(the baryon charge of a quark is taken to be 1/3). The Lagrangian is invariant under $\mathrm{U}(1)_\mathrm{B}$ gauge transformations

$$q \to e^{i\beta/3} q, \tag{2a}$$

$$B_\mu \to B_\nu + \partial_\mu \beta, \tag{2b}$$

The QCD Lagrangian is also invariant under Lorentz transformations, assuming B_ν transforms as a vector.

Let us assume that the baryon number symmetry is spontaneously broken. The low-energy dynamics then contains a NG boson φ_B, which is the $\mathrm{U}(1)_\mathrm{B}$ phase of the condensate. This mode is the superfluid phonon and exists in both the CFL phase and in nuclear matter with nucleon pairing. The transformation property of φ_B can be found from Eq. (2a),

$$\varphi_\mathrm{B} \to \varphi_\mathrm{B} + 2\beta \tag{3}$$

where the factor 2 is the baryon charge of the $\mathrm{U}(1)_\mathrm{B}$ breaking order parameter (assumed to have the baryon charge of a Cooper pair of two baryons). We also assume the there exist a neutral NG boson which comes from breaking of a chiral symmetry. This can be a π^0, η or η' boson. To keep our discussion general, we introduce the current which creates this boson,

$$j^\mathrm{A}_\mu = \frac{1}{2}\sum_{i=1}^{N_\mathrm{f}} Q^5_i\, \bar{\psi}_i \gamma_\mu \gamma^5 \psi_i \tag{4}$$

For example, for π^0, $Q^5_u = -Q^5_d = 1$, $Q^5_s = 0$, and for η', $Q^5_u = Q^5_d = Q^5_s = 1$. This current generates a chiral transformation,

$$\psi_i \to \exp\Big(\frac{i}{2}\alpha Q^5_i \gamma^5\Big)\psi_i\,. \tag{5}$$

The NG boson is characterized by a phase φ_A, which transforms under (5) as

$$\varphi_\mathrm{A} \to \varphi_\mathrm{A} + q_\mathrm{A}\alpha\,, \tag{6}$$

where q_A is a number characterizing the axial charge of the condensate. The factor $\frac{1}{2}$ was introduced in Eqs. (4)

and (5) so that the $U(1)_A$ charge of the chiral condensate $\langle q_L \bar{q}_R \rangle$ is one. In color superconducting phases, the current will be normalized so that $q_A = 2$.

The effective low-energy description must respect the $U(1)_B$ gauge symmetry and be invariant under the gauge transformations (2) and (3). Therefore, in the effective Lagrangian the derivative $\partial_\mu \varphi_B$ should always appear in conjunction with B_μ to make a covariant derivative $D_\mu \varphi_B = \partial_\mu \varphi_B - 2B_\mu$. Vice versa, each occurrence of B_μ has to accompanied by a $-\frac{1}{2}\partial_\mu \varphi_B$. The effective Lagrangian should be relativistically invariant before the replacement $B_\mu \to (\mu, \vec{0})$ is made [7].

In the absence of background fields, neglecting the quark masses, the chiral current (4) is conserved if $\sum_i Q_i^5 = 0$. The conservation is violated for $\sum_i Q_i^5 \neq 0$ by instanton effects, but at sufficiently high density such effects are suppressed. In the presence of the background electromagnetic and $U(1)_B$ fields, the conservation of (4) is violated by triangle anomalies:

$$\begin{aligned}\partial^\mu j_\mu^A = -\frac{1}{16\pi^2}\big(&e^2 C_{A\gamma\gamma} F^{\mu\nu}\tilde{F}_{\mu\nu} - 2eC_{AB\gamma} B^{\mu\nu}\tilde{F}_{\mu\nu} \\ &+ C_{ABB} B^{\mu\nu}\tilde{B}_{\mu\nu}\big)\end{aligned} \tag{7}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$; $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$, $\tilde{B}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}B^{\alpha\beta}$. The coefficients C's in Eq. (7) are given by

$$\begin{aligned}C_{A\gamma\gamma} = 3\sum\nolimits_i Q_i^5 (Q_i)^2, \quad C_{AB\gamma} = \sum\nolimits_i Q_i^5 Q_i, \\ C_{ABB} = \frac{1}{3}\sum\nolimits_i Q_i^5\end{aligned} \tag{8}$$

The effective theory has to reproduce the anomaly relation (7). In the effective theory $\partial_\mu j_\mu^A$ can be found from the change of the action under the transformations (5) and (6): $\delta S = -\int d^4x\, \partial^\mu \alpha\, j_\mu^A = \int d^4x\, \alpha\, \partial^\mu j_\mu^A$. From the condition of anomaly matching one can deduce that the effective Lagrangian contains the following terms,

$$\begin{aligned}\mathscr{L}_{\rm anom} = \frac{1}{8\pi^2 q_A}&\partial_\mu \varphi_A (e^2 C_{A\gamma\gamma} A_\nu \tilde{F}^{\mu\nu} \\ &- 2eC_{AB\gamma} B_\nu \tilde{F}^{\mu\nu} + C_{ABB} B_\nu \tilde{B}^{\mu\nu})\end{aligned} \tag{9}$$

According to our previous discussion, each B_μ has to come with $-\frac{1}{2}\partial_\mu \varphi_B$. Replacing $B_\mu \to B_\mu - \frac{1}{2}\partial_\mu \varphi_B$, and setting afterward $B_\mu = \mu n_\mu$, we find

$$\begin{aligned}\mathscr{L}_{\rm anom} = \frac{1}{8\pi^2 q_A}&\partial_\mu \varphi_A \Big[e^2 C_{A\gamma\gamma} A_\nu \tilde{F}^{\mu\nu} \\ &- 2eC_{AB\gamma}\left(\mu n_\nu - \tfrac{1}{2}\partial_\nu \varphi_B\right)\tilde{F}^{\mu\nu} \\ &- \tfrac{1}{2} C_{ABB}\, \epsilon^{\mu\nu\alpha\beta}\left(\mu n_\nu - \tfrac{1}{2}\partial_\nu \varphi_B\right)\partial_\alpha \partial_\beta \varphi_B\Big]\end{aligned} \tag{10}$$

The term proportional to $C_{A\gamma\gamma}$ in Eq. (10) describes two-photon decays like $\pi^0 \to 2\gamma$ and $\eta' \to 2\gamma$. Such processes occur already in the vacuum. At finite density $\pi^0 \to 2\gamma$ was considered previously in Ref. [2] using a different technique, while $\eta' \to 2\gamma$ was considered in Ref. [3] using the technique adopted in this paper. We do not consider these terms here. The new terms are the ones proportional to $C_{AB\gamma}$ and C_{ABB}.

At first sight, these new terms either vanish identically or are full derivatives and cannot have any physical effect. This is true when the NG fields φ_A and φ_B are small fluctuations from zero. In particular, π^0 does not decay to a phonon and a photon. However, as $\varphi_{A,B}$ are periodic variables, the action can be nonzero if either or both variables make a full 2π rotation. This occurs in the presence of topological defects like vortices or domain walls.

2SC and CFL phases.—Our discussion so far has been rather general. We now specialize ourselves on two specific color superconducting phases: the two-flavor color superconducting (2SC) and the CFL phases, and discuss the topological defects in these phases.

In the 2SC phase, baryon number is not spontaneously broken, hence the field φ_B does not exist. On the other hand, the $U(1)_A$ symmetry is broken by a field Σ constructed from diquark condensates. There is a single $U(1)_A$ NG boson η which corresponds to the $U(1)_A$ current $j_\mu^A = \frac{1}{2}\bar{\psi}\gamma^\mu\gamma^5\psi$ ($Q_u^5 = Q_d^5 = 1$). Σ has charge 2 under $U(1)_A$: $q_A = 2$. If the η mass is sufficiently small, there are metastable domain walls where φ_A changes by 2π [4]. The boundary of such walls are closed η vortices. The coefficients C's are

$$C_{A\gamma\gamma} = \frac{5}{3}, \quad C_{AB\gamma} = \frac{1}{3}, \quad C_{ABB} = \frac{2}{3} \tag{11}$$

In the CFL phase, the gauge-invariant order parameters are also constructed from diquarks,

$$X^{ai} = \epsilon^{ijk}\epsilon^{abc}\epsilon^{\alpha\beta}(q_\alpha^{jb} q_\beta^{kc})^* \tag{12a}$$

$$Y^{ai} = \epsilon^{ijk}\epsilon^{abc}\epsilon^{\dot\alpha\dot\beta}(q_{\dot\alpha}^{jb} q_{\dot\beta}^{kc})^* \tag{12b}$$

as $\Sigma = X^\dagger Y$ which is a 3×3 matrix which breaks chiral and $U(1)_A$ symmetries. One can also construct, e.g., $W = \det X\, {\rm Tr}\,(X^{-1}Y)$ which breaks the baryon number. In terms of quark fields, $\Sigma \sim \bar{q}_L^2 q_R^2$, and $W \sim \bar{q}_L^4 \bar{q}_R^2$.

Obviously, the CFL phase has baryon vortices, see Ref. [3] for details. In fact, a rotating CFL core of a star has to be threaded by a vortex lattice. There are also $U(1)_A$ domain walls [4]. The wall is actually not a pure $U(1)_A$ wall due to the mixing of η and η'. If one parametrizes the three neutral axial NG bosons by $\Sigma = {\rm diag}\,(e^{i\varphi_1}, e^{i\varphi_2}, e^{i\varphi_3})$ then φ_3 is the lightest boson. It is generated by the current

$$j_\mu^A = \frac{1}{2}\left(\bar{u}\gamma_\mu\gamma^5 u + \bar{d}\gamma_\mu\gamma^5 d - \bar{s}\gamma_\mu\gamma^5 s\right), \tag{13}$$

i.e., $Q_u^5 = Q_d^5 = -Q_s^5 = 1$. The transformation generated by j_μ^A leaves φ_1 and φ_2 unchanged, and changes only φ_3, with $q_A = 2$. The coefficients C's are

$$C_{A\gamma\gamma} = \frac{4}{3}, \quad C_{AB\gamma} = \frac{2}{3}, \quad C_{ABB} = \frac{1}{3} \tag{14}$$

In the CFL phase, the photon is mixed with a component of the gluon field. For colorless objects the effect of this mixing is small if the electromagnetic coupling is much smaller than the strong coupling. We will neglect this effect in the future.

Nuclear matter—The anomalous Lagrangian in nuclear matter can be derived in a similar fashion. We assume that neutrons as well as protons form a superfluid which is characterized by the phases φ_n and φ_p correspondingly. We also allow for the neutrons and the protons to have different chemical potentials, μ_n and μ_p respectively. The NG boson we consider is the π^0, which is parametrized by the phase φ_π. The axial current is normalized so that $q_{\rm A} = 1$; so φ_π is related to the canonically normalized π^0 field as $\varphi_\pi = \pi^0/f_\pi$, $f_\pi \approx 93$ MeV in the vacuum. The following Lagrangian is obtained from Eq. (10),

$$\begin{aligned}\mathscr{L}_{\rm anom} = \frac{\partial_\mu\varphi_\pi}{8\pi^2}\Big[&e^2 A_\nu \tilde{F}^{\mu\nu} + e\epsilon^{\mu\nu\alpha\beta}A_\nu\partial_\alpha\partial_\beta\varphi_p \\ &- \epsilon^{\mu\nu\alpha\beta}\left(\mu_p n_\nu - \tfrac{1}{2}\partial_\nu\varphi_p\right)\partial_\alpha\partial_\beta\varphi_p \\ &- \epsilon^{\mu\nu\alpha\beta}\left(\mu_n n_\nu - \tfrac{1}{2}\partial_\nu\varphi_n\right)\partial_\alpha\partial_\beta\varphi_n\Big]\end{aligned} \tag{15}$$

Armed with the effective anomalous Lagrangians (10) and (15), we now can discuss physical consequences of quantum anomalies in densed matter. We do not try to exhaust all effects, but we would like to mention a few representative ones.

Axial current on a superfluid vortex.—To start, let us consider a superfluid (baryon) vortex. For definiteness, we assume the CFL phase, but many details are applicable for nuclear matter as well. Let the vortex be parallel to the z axis and located at $x = y = 0$. Around the vortex $\varphi_{\rm B}$ changes by 2π, and in the center of the vortex it is ill-defined. Outside the core $\epsilon^{\mu\nu\alpha\beta}\partial_\alpha\partial_\beta\varphi_{\rm B} = 0$, and the term in $\mathscr{L}_{\rm anon}$ that contains $\varphi_{\rm B}$ vanishes. However, at the vortex core, as usual, $(\partial_x\partial_y - \partial_y\partial_x)\varphi_{\rm B} = 2\pi\delta^2(x_\perp)$. The action becomes

$$S_{\rm anom} = \frac{\mu}{12\pi}\int dt\, dz\, \partial_z\varphi_{\rm A} \tag{16}$$

where the integral is a linear integral along the vortex line. Now let us recall that the axial current $J^{\rm A}_\mu$ is obtained, from Noether's theorem, by differentiating the action with respect to $\partial_\mu\varphi_\pi$. One sees immediately that *there is an axial current running on the superfluid vortex*, with a magnitude of $\mu/(12\pi)$. This current, naturally, is coupled to the Z boson.

From Eq. (15) one finds an axial current on a neutron superfluid vortex in nuclear matter. This current can be understood as a nonzero density of nucleon spin on the vortex.

Axial boson–photon coupling on a baryon vortex.—In the CFL phase there is the following term in the effective Lagrangian,

$$\partial_\mu\varphi_{\rm A}\partial_\nu\varphi_{\rm B}\tilde{F}^{\mu\nu} \sim \varphi_{\rm A}\partial_\mu\partial_\nu\varphi_{\rm B}\tilde{F}^{\mu\nu} \tag{17}$$

This term is zero for topologically trivial field configurations but does not vanish in the presence of vortices. For a baryon vortex located at $x = y = 0$ we again set $(\partial_x\partial_y - \partial_y\partial_x)\varphi_{\rm B} = 2\pi\delta^2(x_\perp)$, and the Lagrangian term becomes $\delta^2(x_\perp)\varphi_{\rm A}F_{03}$. We thus find that there is a linear coupling of neutral NG bosons with the electric field in the vortex core. Therefore, NG boson (π^0, η, η') striking the vortex line can be converted to one photon and vibrations of the vortex line. In the absense of vortices these bosons can only decay into two photons.

Similar conclusion can be reached for π^0 striking the proton superfluid vortex in nuclear matter.

Magnetization of axial domain walls.—Let us consider an axial domain wall in an external magnetic field. Such domain walls exist at very high densities where instanton effects are suppressed [4]. When the baryon field B_ν is treated as a background, $B_\nu = (\mu, \vec{0})$, the following term is present in the anomaly Lagrangian:

$$\mathscr{L}_{{\rm AB}\gamma} = \frac{eC_{{\rm AB}\gamma}\mu}{8\pi^2}\vec{B}\cdot\vec{\nabla}\varphi_{\rm A} \tag{18}$$

where we have set $q_{\rm A} = 2$ and $\vec{B}$ is the magnetic field. Consider now a $\mathrm{U}(1)_{\rm A}$ domain wall streched along the xy directions. On the wall $\varphi_{\rm A}$ has a jump by 2π. Now turn on a magnetic field perpendicular to the wall, i.e., along the z direction. Equation (18) implies that the energy is changed by a quantity proportional to BS, where S is the area of the domain wall. This means that *the domain wall is magnetized*, with a finite magnetic moment per unit area equal to $eC_{{\rm AB}\gamma}\mu/(4\pi)$. The magnetic moment is directed perpendicularly to the domain wall. For the 2SC and CFL phases the magnetic moment per unit area is $e\mu/(12\pi)$ and $e\mu/(6\pi)$, respectively.

Currents on axial vortices.—The same effect can be looked at from a different perspective. One rewrites Eq. (18) into the following form,

$$\mathscr{L}_{\rm anom} = \frac{eC_{{\rm AB}\gamma}\mu}{4\pi^2}\,\epsilon_{ijk}A_i\,\partial_j\partial_k\varphi_{\rm A} \tag{19}$$

Since $\epsilon_{ijk}\partial_j\partial_k\varphi_{\rm A} \sim 2\pi\delta^2(x_\perp)$ on the vortex core, the action can be written as a line integral along the vortex,

$$S_{\rm anom} = \frac{eC_{{\rm AB}\gamma}\mu}{2\pi}\int d\vec{\ell}\cdot\vec{A} \tag{20}$$

which means that *there is an electric current running along the core of the axial vortex*, which is similar to the axial current on baryon vortices. The magnitude of the current is given by

$$j^{\rm em} = \frac{eC_{{\rm AB}\gamma}\mu}{2\pi} \tag{21}$$

Taking $\mu \sim 1.5$ GeV, in the CFL phase this is about 40 kA.

Naturally, the electromagnetic current running along a closed vortex loop generates a magnetic moment equal

to $\frac{1}{2}jS$, where S is the area of the surface enclosed by the loop. This is exactly what we found in Eq. (18). A large vortex loop, therefore, has a magnetic moment that can be interpreted as created by the current running along the loop, *or* as the total magnetization of the domain wall stretched on the loop. The two pictures come from the same term in the anomaly Lagrangian.

An axial vortex loop also carries angular momentum. To see that, let us place a vortex loop inside a rotating quark matter. We thus put in Eq. (9) $B_\nu = \mu v_\nu$ where v_ν is the local velocity and $\partial_i v_j - \partial_j v_i = 2\epsilon_{ijk}\omega_k$. The ABB term in Eq. (9) reads

$$\mathscr{L}_{\mathrm{ABB}} = \frac{C_{\mathrm{ABB}}\mu^2}{8\pi^2}\vec{\omega}\cdot\vec{\nabla}\varphi_{\mathrm{A}} \tag{22}$$

which implies that the domain wall is characterized by a constant angular momentum per unit area, equal to $C_{\mathrm{ABB}}\mu/(4\pi)$. As in the case of the magnetic moment, this angular momentum can alternatively be interpreted as being carried by an energy (mass) current running along the vortex surrounding the wall.

As is well known, for a relativistic system in order to have a nonzero value for the angular momentum $M^{ij} = \int d^3x\,(x^iT^{0j} - x^jT^{0i})$ one needs a time dependence such that $T^{0i} \neq 0$. Such time dependece is automatically appears if a system put into a medium [8]. In our case the construction of Ref. [8] is automatically realized due to a nonzero value of μ in our system.

Conclusions.—We have seen that the quantum anomalies, especially those involving the baryon current, lead to new and extremely unusual effects in high-density matter. The effects appear when topological objects (vortices, domain walls) are present. Some effects may have consequences for the physics of compact objects, which are to be explored. Of particular interest is the finding that baryon vortices in the CFL phase and in nuclear matter carry weak neutral current, which interact with neutrinos and may affect the cooling process of neutron stars.

Some of the effects discussed in this paper have precursors previously discussed in the literature. The currents flowing on vortices are reminiscent of Witten's superconducting strings [9] and of zero fermion modes on a vortex [10]. Previously, Witten's construction has been considered for dense matter [11–13]. In our case, the currents on vortices appear in a much more direct fashion.

It should be said that there are many more physical effects of anomalies that have not been considered in this paper. They be retrieved quite easily from the Lagrangian terms derived here. One particularly interesting effect is that there is an electric charge located at the junction of a $\mathrm{U}(1)_{\mathrm{A}}$ domain wall and a baryon vortex, and that this charge is fractional.

All results in this paper have been derived from the anomaly terms of the effective Lagrangian and do not rely on details of the microscopic QCD Lagrangian, except for anomaly relations in the latter. On the other hand, it should be possible to understand the microscopic origin of the vortex currents found here. We plan to return to this question in the future.

We have concentrated in this paper on a few phases of dense matter (neutron superfluids, 2SC and CFL phases). The results should be extendable to other phases, e.g., phases with kaon and eta condensation [14].

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